JEM013 - Game Theory

Seminar 2

Iterated Weak Dominance, Nash Equilibrium, Mixed Strategy

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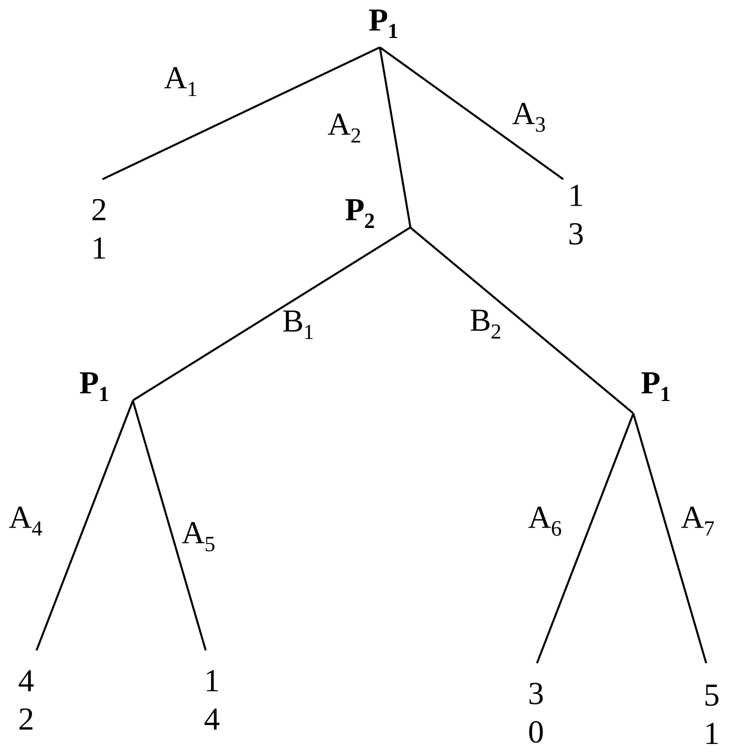
Homework 1.1: Consider the game



1.1.1 **Find backward-induction outcome** of this game. Write the payoffs which players receive at the end of the game.

1.1.2 **Write** down all possible **strategies of type B** for both players

1.1.3 **Write** down all possible **strategies of type A** for both players.

1.1.4 Which strategies (of type B) are consistent with backwards induction in this game?

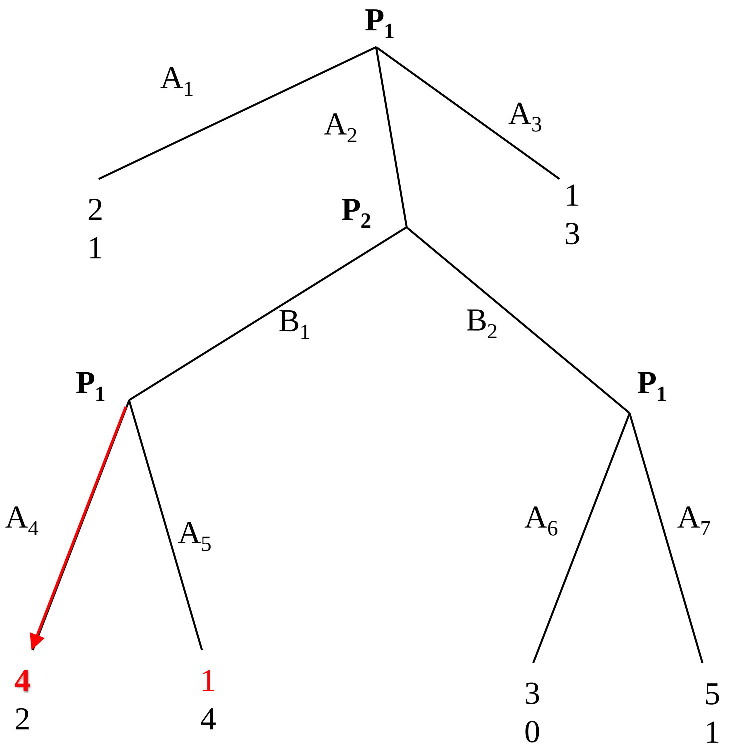
1.1.5 **Write this game in normal (strategic) form** using the strategies of type B and type A.

1.1.6 Find **pure-strategy Nash equilibri**a in this game.

1.1.7 **List**, for each player, the **set of strategies consistent with rationalizability.**

1.1.8 **List,** for each player, the **set of strategies remaining after the iterated deletion of weakly dominated strategies.**

Homework 1.1: Consider the game



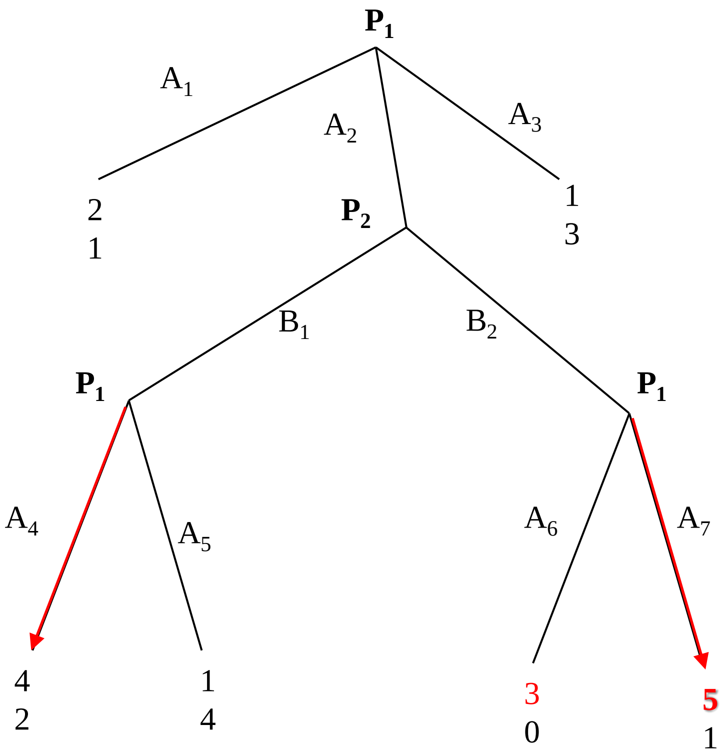
Perfect-Information Games

Backward induction

Homework 1.1: Consider the game

Perfect-Information Games

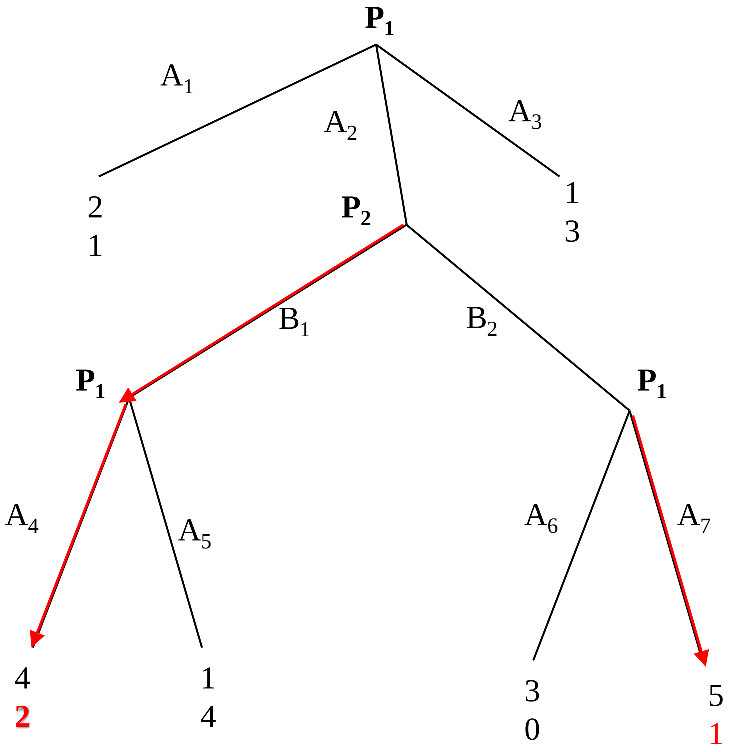
Backward induction



Homework 1.1: Consider the game

Perfect-Information Games

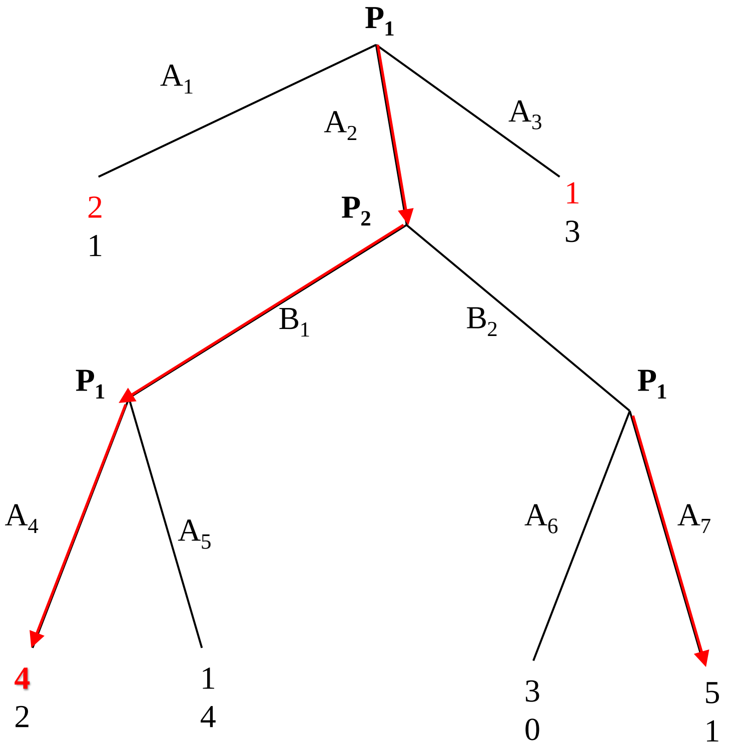
Backward induction



Homework 1.1: Consider the game

Perfect-Information Games

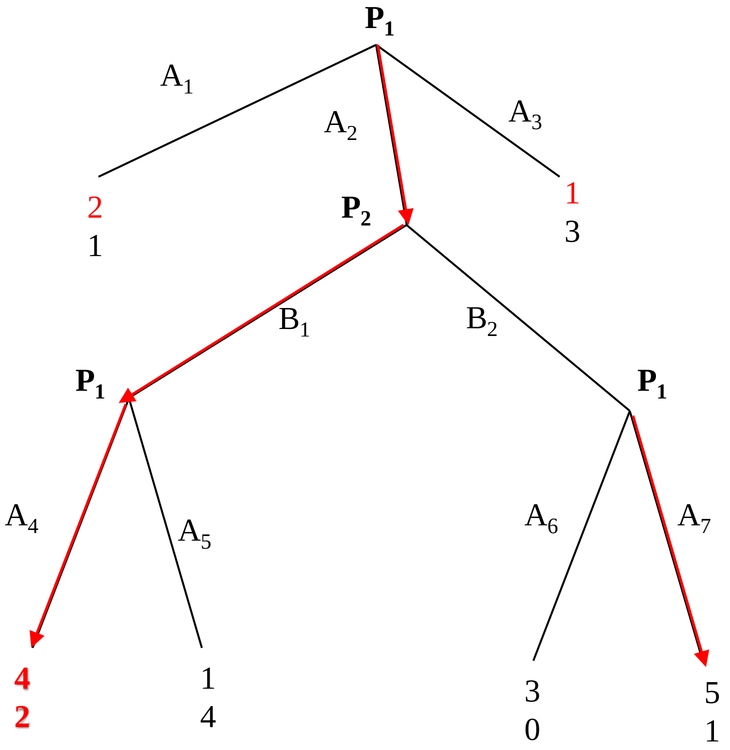
Backward induction



Homework 1.1: Consider the game

Perfect-Information Games

Backward induction



1.1.1 **Find backward-induction outcome** of this game. Write the payoffs which players receive at the end of the game.

BI Outcome: P1→A_2A_4 ; P2→B_1

BI strategies profile: $(A_2A_4\,A_7\,;\,B_1)$

Payoffs (4, 2)

Homework 1.1: Consider the game

- All possible strategies of type B for both players:

P1:

$A_1A_4A_6$
$A_1A_4A_7$
$A_1A_5A_6$
$A_1A_5A_7$
$A_2A_4A_6$
$A_2A_4A_7$
$A_2A_5A_6$
$A_2A_5A_7$
$A_3A_4A_6$
$A_3A_4A_7$
$A_3A_5A_6$
$A_3A_5A_7$

P2:

B_1
B_2

- All possible strategies of type A for both players

P1:

A_1
$A_2A_4A_6$
$A_2A_4A_7$
$A_2A_5A_6$
$A_2A_5A_7$
A_3

P2:

B_1
B_2

1.1.5 **Write this game in normal (strategic) form** using the strategies of type B and type A.

P_2

		B_1	3. ~~B_2~~	
	~~$A_1A_4A_6$~~	~~2, 1~~	~~2, 1~~	1.
	~~$A_1A_4A_7$~~	~~2, 1~~	~~2, 1~~	1.
	~~$A_1A_5A_6$~~	~~2, 1~~	~~2, 1~~	1.
	~~$A_1A_5A_7$~~	~~2, 1~~	~~2, 1~~	1.
P_1	$A_2A_4A_6$	4, 2	~~3, 0~~	
	$A_2A_4A_7$	4, 2	~~5, 1~~	
	~~$A_2A_5A_6$~~	~~1, 4~~	~~3, 0~~	4.
	~~$A_2A_5A_7$~~	~~1, 4~~	~~5, 1~~	4.
	~~$A_3A_4A_6$~~	~~1, 3~~	~~1, 3~~	2.
	~~$A_3A_4A_7$~~	~~1, 3~~	~~1, 3~~	2.
	~~$A_3A_5A_6$~~	~~1, 3~~	~~1, 3~~	2.
	~~$A_3A_5A_7$~~	~~1, 3~~	~~1, 3~~	2.

Strategic form with type B strategies

P_2

		B_1	B_2
	A_1	2, 1	2, 1
	$A_2A_4A_6$	4, 2	3, 0
P_1	$A_2A_4A_7$	4, 2	5, 1
	$A_2A_5A_6$	1, 4	3, 0
	$A_2A_5A_7$	1, 4	5, 1
	A_3	1, 3	1, 3

Strategic form with type A strategies

1.1.6 Find **pure-strategy Nash equilibria** in this game.

NE: ($A_2A_4A_6$, B_1) payoffs (4, 2)

NE: ($A_2A_4A_7$, B_1) payoffs (4, 2)

1.1.7 **List**, for each player, the **set of strategies consistent with rationalizability.**

$R_1=\{A_2A_4A_6 , A_2A_4A_7 \}$; $R_2=\{B_1\}$

1.1.8 **List,** for each player, the **set of strategies remaining after the iterated deletion of weakly dominated strategies.**

$I_1=\{A_2A_4A_6 , A_2A_4A_7 \}$, $I_2= \{B_1\}$

Homework 1.2: Consider the game

		P_2			
		b_1	b_2	b_3	b_4
P_1	a_1	6; 1	1; 2	5; 4	-1; -2
	a_2	5; 5	4; 2	2; 1	0; -1
	a_3	1; 2	0; 6	2; 3	2; -3
	a_4	0;-1	-4; 0	0; 1	-6; 0

1.2.1 **List**, for each player, the **set of strategies consistent with rationalizability** (When you eliminate any strategy, explain clearly why you eliminate it.)

1.2.2 **Write** this game in the **extensive form (game tree).**

Homework 1.2: Consider the game

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. b_4 is SDed by b_3
$(A_1(1); A_2(1))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3\})$ | P_2 is R

2. a_4 is SDed by a_1 and a_3
3. a_3 is SDed by a_1
$(A_1(3); A_2(3))=(\{a_1,a_2\}, \{b_1, b_2, b_3\})$ | P_1 k P_2 is R,
P_1 is R

4. b_2 is not rationalizable
$(A_1(4); A_2(4))=(\{a_1,a_2\}, \{b_3, b_4\})$ | P_2 k P_1 k P_2 is R,
P_2 k P_1 is R

5. a_2 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 is R,
P_1 k P_2 k P_1 is R

P_2

P_1

		b_1 6.	b_2	b_3	b_4	
p	a_1	6; 1	1; 2	5; 4	-1; -2	1.
1-p	a_2	5; 5	4; 2	2; 1	0; -1	5.
	a_3	1; 2	0; 6	2; 3	2; -3	3
	a_4	0;-1	-4; 0	0; 1	-6; 0	2.
			4.		1.	

b_2-?
$EU_2(b_1)=5-4p$
$EU_2(b_2)=2$
$EU_2(b_3)=1+3p$

$EU_2(b_1)= EU_2(b_2) \Rightarrow p=3/4$
$EU_2(b_2)= EU_2(b_3) \Rightarrow p=1/3$

$p>1/3 \Rightarrow b_1>b_2$, $p<3/4 \Rightarrow b_3>b_2$
=> no beliefs that b_2 is optimal

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$
$R_1=\{a_1\}$, $R_2= \{b_3\}$

Common knowledge of rationality

each player is rational,
each player knows that each player is rational,
each player knows that each player knows that each player is rational,
...
and so forth ad infinitum.

Definition of rationalizable actions

Let $(A_1(0), A_2(0), \ldots, A_N(0))=(A_1, A_2, \ldots, A_N)$
Then for all $t\geq 1$, for all players k,
let $A_k(t)=\{a \in A_k(t-1)|$ a maximizes k's EU given some beliefs over actions in $\{A_j(t-1)\}_{j\neq k}\}$
Then we say that the set of RATIONALIZABLE actions for player k is
$R_k=\lim A_k(t)$ at $t\rightarrow\infty$.

Lemma

If for **ALL** k $A_k(t+1)=A_k(t)$, then for all k $R_k=A_k(t)$

Exercise 1

P_2

		b_1	b_2	b_3	b_4
P_1	a_1	1; 0	3;-1	5; 0	-1; 2
	a_2	9;-1	8;-2	6; 2	0; 1
	a_3	5;-3	6;-4	2; 0	2; 3
	a_4	9;-1	4; 0	0; 1	6; 0

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$

$R_1=\{a_2\}$, $R_2= \{b_3\}$

P_2

P_1		b_1	b_2	b_3	b_4	
	~~a_1~~	~~1; 0~~	~~3;-1~~	~~5; 0~~	~~-1; $\underline{2}$~~	1.
	a_2	$\underline{9}$;-1	$\underline{8}$;-2	$\underline{6}$; $\underline{2}$	0; 1	
	a_3	5;-3	6;-4	2; 0	2; $\underline{3}$	
	a_4	$\underline{9}$;-1	4; 0	0; $\underline{1}$	$\underline{6}$; 0	

a_3-?
$EU_1(a_2)=6q$
$EU_1(a_3)=2$
$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$
$EU_1(a_3)= EU_1(a_4) \Rightarrow q=2/3$

$q>1/3$ $a_2>a_3$, $q<2/3$ $a_4>a_3$
=> no beliefs that a_3 is optimal

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

			P_2			
		b_1	b_2	b_3	b_4	
	a_1	~~1; 0~~	~~3;-1~~	~~5; 0~~	~~-1; $\underline{2}$~~	1.
P_1	a_2	~~$\underline{9}$;-1~~	$\underline{8}$;-2	$\underline{6}$; $\underline{2}$	0; 1	
	a_3	~~5;-3~~	6;-4	2; 0	2; $\underline{3}$	
	a_4	~~$\underline{9}$;-1~~	4; 0	0; $\underline{1}$	$\underline{6}$; 0	
		2.				

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$

$R_1=\{a_2\}$, $R_2= \{b_3\}$

a_3-?
$EU_1(a_2)=6q$
$EU_1(a_3)=2$
$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$
$EU_1(a_3)= EU_1(a_4) \Rightarrow q=2/3$

$q>1/3$ $a_2>a_3$, $q<2/3$ $a_4>a_3$
=> no beliefs that a_3 is optimal

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$

$R_1=\{a_2\}$, $R_2= \{b_3\}$

		P_2				
		b_1	b_2	b_3	b_4	
	a_1	~~1; 0~~	~~3;-1~~	~~5; 0~~	~~-1; $\underline{2}$~~	1.
P_1	a_2	~~$\underline{9}$;-1~~	~~$\underline{8}$;-2~~	$\underline{6}$; $\underline{2}$	0; 1	
	a_3	~~5;-3~~	~~6;-4~~	2; 0	2; $\underline{3}$	
	a_4	~~$\underline{9}$;-1~~	~~4; 0~~	0; $\underline{1}$	$\underline{6}$; 0	
		2.	3.			

a_3-?

$EU_1(a_2)=6q$

$EU_1(a_3)=2$

$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$

$EU_1(a_3)= EU_1(a_4) \Rightarrow q=2/3$

$q>1/3$ $a_2>a_3$, $q<2/3$ $a_4>a_3$

=> no beliefs that a_3 is optimal

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

			P_2			
		b_1	b_2	b_3	b_4	
	a_1	1; 0	3;-1	5; 0	-1; $\underline{2}$	1.
	a_2	$\underline{9}$;-1	$\underline{8}$;-2	$\underline{6}$; $\underline{2}$	0; 1	
P_1	a_3	5;-3	6;-4	2; 0	2; $\underline{3}$	4.
	a_4	$\underline{9}$;-1	4; 0	0; $\underline{1}$	$\underline{6}$; 0	
		2.	3.	q	1-q	

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$

$R_1=\{a_2\}$, $R_2= \{b_3\}$

a_3-?
$EU_1(a_2)=6q$
$EU_1(a_3)=2$
$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$
$EU_1(a_3)= EU_1(a_4) \Rightarrow q=2/3$

$q>1/3$ $a_2>a_3$, $q<2/3$ $a_4>a_3$
=> no beliefs that a_3 is optimal

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

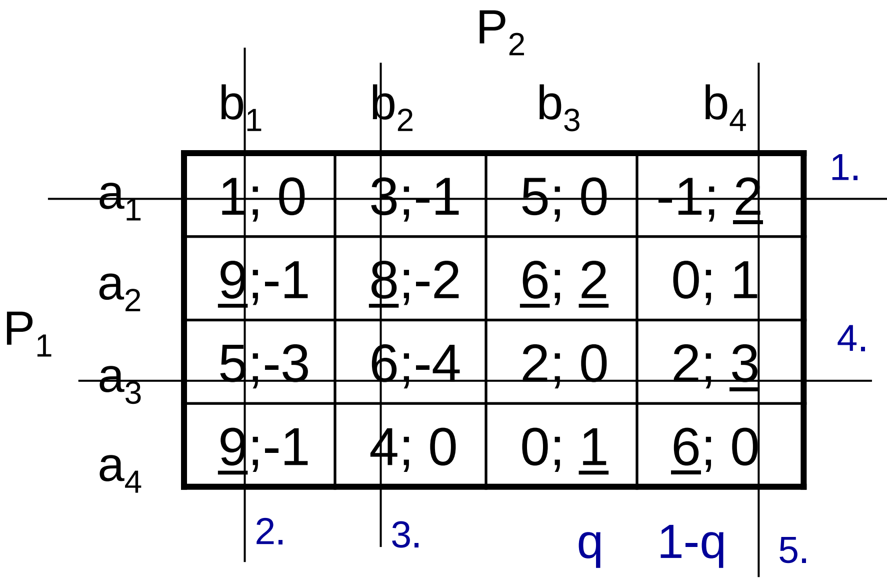
6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t) \Rightarrow R_k=A_k(t)$

$R_1=\{a_2\}, R_2= \{b_3\}$



	b_1	b_2	b_3	b_4
a_1	1; 0	3;-1	5; 0	-1; $\underline{2}$
a_2	$\underline{9}$;-1	$\underline{8}$;-2	$\underline{6}$; $\underline{2}$	0; 1
a_3	5;-3	6;-4	2; 0	2; $\underline{3}$
a_4	$\underline{9}$;-1	4; 0	0; $\underline{1}$	$\underline{6}$; 0

a_3-?
$EU_1(a_2)=6q$
$EU_1(a_3)=2$
$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$
$EU_1(a_3)= EU_1(a_4) \Rightarrow q=2/3$

$q>1/3\ a_2>a_3,\ q<2/3\ a_4>a_3$
=> no beliefs that a_3 is optimal

Exercise 1

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_1 is SDed by a_2

$(A_1(1); A_2(1))=(\{a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$ | P_1 is R

2. b_1 is SDed by b_4
3. b_2 is SDed by b_3

$(A_1(2); A_2(2))=(\{a_2,a_3,a_4\}, \{b_3, b_4\})$ | P_2 k P_1 is R,
P_2 is R

4. a_3 is not rationalizable

$(A_1(3); A_2(3))=(\{a_2,a_4\}, \{b_3, b_4\})$ | P_1 k P_2 k P_1 is R,
P_1 k P_2 is R

5. b_4 is SDed by b_3

$(A_1(4); A_2(4))=(\{a_2,a_4\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 is R

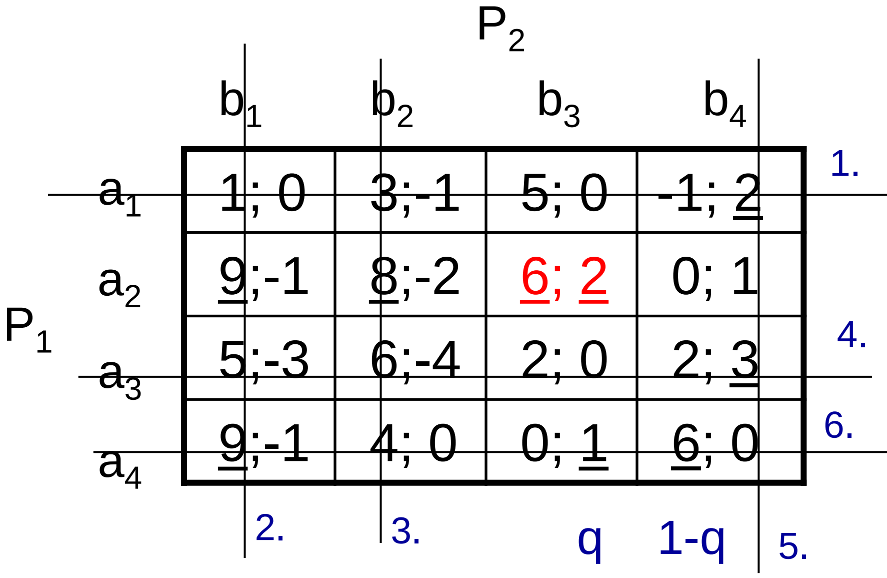
6. a_4 is SDed by a_2

$(A_1(5); A_2(5))=(\{a_2\}, \{b_3\})$ | P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_1 k P_2 k P_1 k P_2 is R

$(A_1(6); A_2(6))=(\{a_2\}, \{b_3\})$ | P_2 k P_1 k P_2 k P_1 k P_2 k P_1 is R,
P_2 k P_1 k P_2 k P_1 k P_2 is R

For each player k $A_k(t+1)=A_k(t)$ => $R_k=A_k(t)$

$R_1=\{a_2\}$, $R_2= \{b_3\}$



P_2

		b_1	b_2	b_3	b_4
P_1	a_1	1; 0	3;-1	5; 0	-1; $\underline{2}$
	a_2	$\underline{9}$;-1	$\underline{8}$;-2	$\underline{6}$; $\underline{2}$	0; 1
	a_3	5;-3	6;-4	2; 0	2; $\underline{3}$
	a_4	$\underline{9}$;-1	4; 0	0; $\underline{1}$	$\underline{6}$; 0

a_3-?
$EU_1(a_2)=6q$
$EU_1(a_3)=2$
$EU_1(a_4)=6-6q$

$EU_1(a_2)= EU_1(a_3)$ => $q=1/3$
$EU_1(a_3)= EU_1(a_4)$ => $q=2/3$

$q>1/3$ $a_2>a_3$, $q<2/3$ $a_4>a_3$
=> no beliefs that a_3 is optimal

Exercise 2

Is a_2 rationalizable?

		P_2	
		b_1	b_2
P_1	a_1	3; 0	2; 0
	a_2	2; 0	2; 0

Exercise 2

Is a_2 rationalizable?

Yes.

a_2 is the best response, if P_2 plays b_2 with the probability 1.

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{P_2} \\
 & & b_1 & b_2 \\ \cline{3-4}
P_1 & a_1 & \underline{3}; \underline{0} & \underline{2}; \underline{0} \\ \cline{3-4}
 & a_2 & 2; \underline{0} & \underline{2}; \underline{0} \\ \cline{3-4}
\end{array}$$

Exercise 2

Is a_2 rationalizable?

Yes.

a_2 is the best response, if P_2 plays b_2 with the probability 1.

However, if there is an arbitrary small probability, that P_2 plays b_1, a_1 yields higher expected payoffs than a_2 =>

a_2 is weakly dominated by a_1

(1. a_2 is WDed by a_1)

		P_2		
		b_1	b_2	
P_1	a_1	$\underline{3}; \underline{0}$	$\underline{2}; \underline{0}$	
	~~a_2~~	~~$\underline{2}; \underline{0}$~~	~~$\underline{2}; \underline{0}$~~	1.

Solution concept

Imperfect-Information Games

Iterated strict dominance (ISD)

Players iteratively delete strictly dominated strategies

A strategy is **strictly dominated** if there is some alternative strategy, that yields a strictly greater payoff regardless of what the other players do.

1. a_1 is strictly dominated (SDed) by a_2

		P_2		
		b_1	b_2	
P_1	~~a_1~~	~~1 ; 4~~	~~2 ; 3~~	1.
	a_2	3 ; 2	4 ; 2	

Solution concept

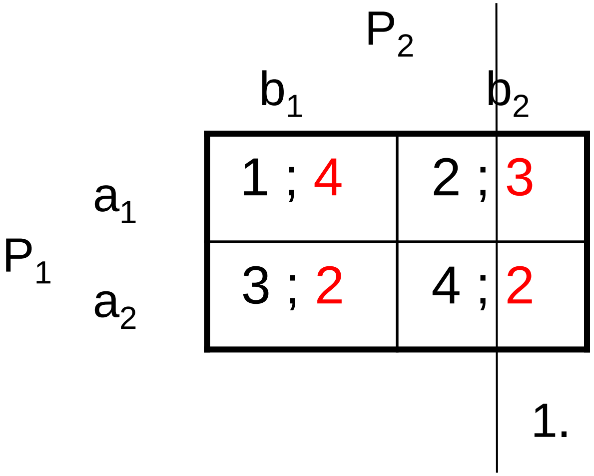
Imperfect-Information Games

Iterated weak dominance (IWD)

Players iteratively delete weakly dominated strategies

A strategy is **weakly dominated** if there is some alternative strategy, that yields a greater or equal payoff and never yielding a lower payoff regardless of what the other players do.

1. b_2 is weakly dominated (WDed) by b_1



		P_2	
		b_1	b_2
P_1	a_1	1 ; 4	2 ; 3
	a_2	3 ; 2	4 ; 2

1.

Solution concept

Rationalizability

ANY beliefs that a strategy is the best response

Let $(A_1(0), A_2(0), \ldots, A_N(0))=(A_1, A_2, \ldots, A_N)$
Then for all $t \geq 1$, for all players k,
let $A_k(t)=\{a \epsilon A_k(t-1)|$ a maximizes k's EU given some beliefs over actions in $\{A_j(t-1)\}_{j \neq k}\}$
Then we say that the set of RATIONALIZABLE actions for player k is
$R_k=\lim A_k(t)$ at $t \rightarrow \infty$.

Iterated weak dominance (IWD)

FULL SUPPORT beliefs that a strategy is the best response

(FULL SUPPORT=strictly positive probabilities for all strategies played by other player)

Let $(A_1(0), A_2(0), \ldots, A_N(0))=(A_1, A_2, \ldots, A_N)$
Then for all $t \geq 1$, for all players k,
let $A_k(t)=\{a \epsilon A_k(t-1)|$ a maximizes k's EU given some full support beliefs over actions in $\{A_j(t-1)\}_{j \neq k}\}$
Then we say that the set of IWD actions for player k is $I_k=\lim A_k(t)$ at $t \rightarrow \infty$.

Exercise 3

		P_2			
		b_1	b_2	b_3	b_4
P_1	a_1	2; 2	2; 2	0; 0	0; 0
	a_2	0; 0	0; 0	1; 1	1; 1
	a_3	1; 2	-1; 0	1; 2	-1; 0
	a_4	-1; 0	0; 1	-1; 0	0; 1

Exercise 3

		P_2				
		b_1	b_2	b_3	b_4	
P_1	a_1	$\underline{2}$; $\underline{2}$	$\underline{2}$; $\underline{2}$	0; 0	0; 0	
	a_2	0; 0	0; 0	$\underline{1}$; $\underline{1}$	$\underline{1}$; $\underline{1}$	
	a_3	1; $\underline{2}$	-1; 0	$\underline{1}$; $\underline{2}$	-1; 0	
	a_4	~~-1; 0~~	~~0; 1~~	~~-1; 0~~	~~0; 1~~	1.

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2
$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1
3. b_4 is WDed by b_3
$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3
$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1
$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$
$I_1=\{a_1\}, I_2= \{b_1\}$

Exercise 3

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2

$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1

3. b_4 is WDed by b_3

$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3

$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1

$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1

$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

P_2

		b_1	b_2	b_3	b_4	
	a_1	$\underline{2}; \underline{2}$	$\underline{2}; \underline{2}$	0; 0	0; 0	
P_1	a_2	0; 0	0; 0	$\underline{1}; \underline{1}$	$\underline{1}; \underline{1}$	
	a_3	1; $\underline{2}$	-1; 0	$\underline{1}; \underline{2}$	-1; 0	
	a_4	-1; 0	0; 1	-1; 0	0; 1	1.
			2.			

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}, I_2= \{b_1\}$

Exercise 3

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2
$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1
3. b_4 is WDed by b_3
$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3
$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1
$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

			P_2		
		b_1	b_2	b_3	b_4
	a_1	$\underline{2}$; $\underline{2}$	$\underline{2}$; $\underline{2}$	0; 0	0; 0
P_1	a_2	0; 0	0; 0	$\underline{1}$; $\underline{1}$	$\underline{1}$; $\underline{1}$
	a_3	1; $\underline{2}$	-1; 0	$\underline{1}$; $\underline{2}$	-1; 0
	a_4	-1; 0	0; 1	-1; 0	0; 1

1.

2.

3.

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}$, $I_2= \{b_1\}$

Exercise 3

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2
$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1
3. b_4 is WDed by b_3
$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3
$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1
$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

P_2

P_1	b_1	b_2	b_3	b_4
a_1	$\underline{2}$; $\underline{2}$	$\underline{2}$; $\underline{2}$	0; 0	0; 0
a_2	0; 0	0; 0	$\underline{1}$; $\underline{1}$	$\underline{1}$; $\underline{1}$
a_3	1; $\underline{2}$	-1; 0	$\underline{1}$; $\underline{2}$	-1; 0
a_4	-1; 0	0; 1	-1; 0	0; 1

1.

2.

3.

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}, I_2= \{b_1\}$

Exercise 3

		P_2				
		b_1	b_2	b_3	b_4	
	a_1	$\underline{2}; \underline{2}$	$\underline{2}; \underline{2}$	0; 0	0; 0	
P_1	a_2	0; 0	0; 0	$\underline{1}; \underline{1}$	$\underline{1}; \underline{1}$	4.
	a_3	$1; \underline{2}$	-1; 0	$\underline{1}; \underline{2}$	-1; 0	
	a_4	-1; 0	0; 1	-1; 0	0; 1	1.
			2.		3.	

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2
$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1
3. b_4 is WDed by b_3
$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3
$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1
$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}$, $I_2= \{b_1\}$

Exercise 3

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2

$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1

3. b_4 is WDed by b_3

$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3

$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

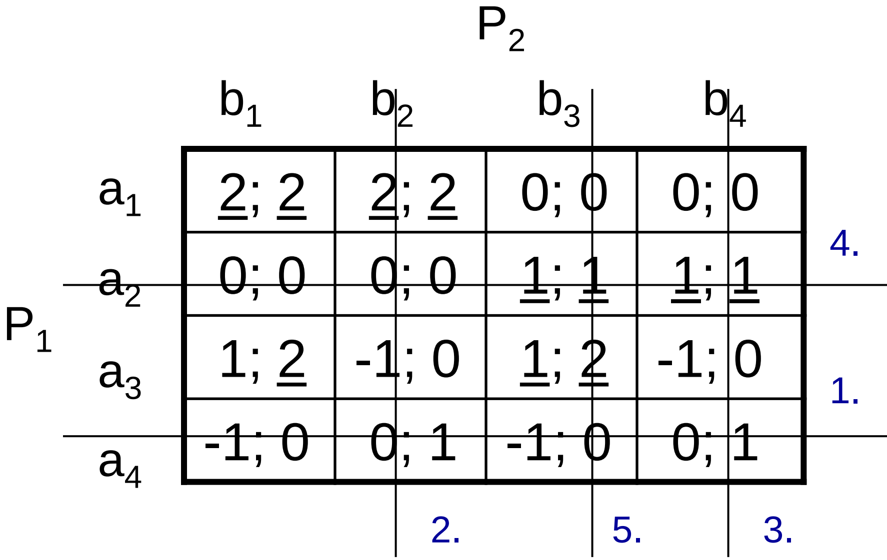
5. b_3 is WDed by b_1

$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1

$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$



	b_1	b_2	b_3	b_4
a_1	$\underline{2}; \underline{2}$	$\underline{2}; \underline{2}$	0; 0	0; 0
a_2	0; 0	0; 0	$\underline{1}; \underline{1}$	$\underline{1}; \underline{1}$
a_3	1; $\underline{2}$	-1; 0	$\underline{1}; \underline{2}$	-1; 0
a_4	-1; 0	0; 1	-1; 0	0; 1

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}, I_2= \{b_1\}$

Exercise 3

$(A_1(0); A_2(0))=(\{a_1,a_2,a_3,a_4\}, \{b_1, b_2, b_3, b_4\})$

1. a_4 is WDed by a_1 and by a_2
$(A_1(1); A_2(1))=(\{a_1, a_2,a_3\}, \{b_1, b_2, b_3, b_4\})$

2. b_2 is WDed by b_1
3. b_4 is WDed by b_3
$(A_1(2); A_2(2))=(\{a_1,a_2,a_3\}, \{b_1, b_3\})$

4. a_2 is WDed by a_3
$(A_1(3); A_2(3))=(\{a_1,a_3\}, \{b_1, b_3\})$

5. b_3 is WDed by b_1
$(A_1(4); A_2(4))=(\{a_1,a_3\}, \{b_1\})$

6. a_3 is SDed by a_1
$(A_1(5); A_2(5))=(\{a_1\}, \{b_1\})$

$(A_1(6); A_2(6))=(\{a_1\}, \{b_1\})$

P_2

P_1

	b_1	b_2	b_3	b_4	
a_1	$\underline{2}; \underline{2}$	$\underline{2}; \underline{2}$	0; 0	0; 0	
a_2	0; 0	0; 0	$\underline{1}; \underline{1}$	$\underline{1}; \underline{1}$	4.
a_3	1; $\underline{2}$	-1; 0	$\underline{1}; \underline{2}$	-1; 0	6.
a_4	-1; 0	0; 1	-1; 0	0; 1	1.
		2.	5.	3.	

For each player k $A_k(t+1)=A_k(t) \Rightarrow I_k=A_k(t)$

$I_1=\{a_1\}, I_2= \{b_1\}$

Does the order of deletion affect the set of strategies that remains in the end ?

		P_2	
		b_1	b_2
P_1	a_1	5; 1	4; 0
	a_2	6; 0	3; 1
	a_3	6; 4	4; 4

Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO
IWD: YES, the order of deletion can affect the outcome

		P_2	
		b_1	b_2
P_1	a_1	5; 1	4; 0
	a_2	6; 0	3; 1
	a_3	6; 4	4; 4

Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO
IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3

		P_2		
		b_1	b_2	
P_1	~~a_1~~	~~5; $\underline{1}$~~	~~$\underline{4}$; 0~~	1.
	a_2	$\underline{6}$; 0	3; $\underline{1}$	
	a_3	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$	

Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3

2. b_1 is WDed by b_2

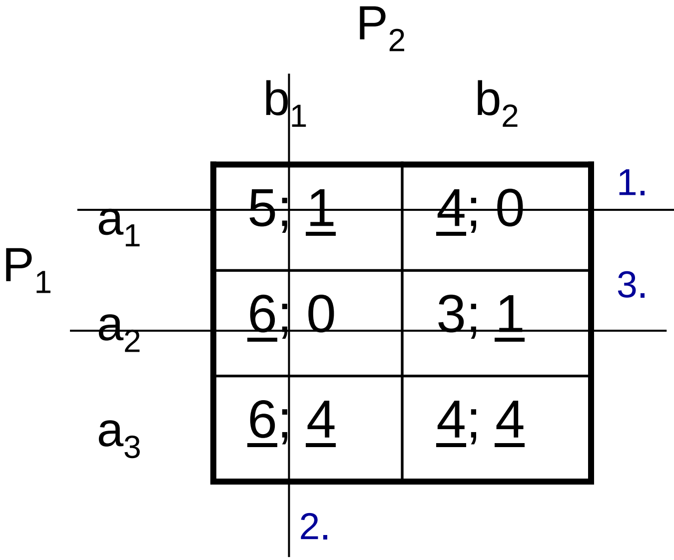
		P_2		
		~~b_1~~	b_2	
P_1	~~a_1~~	~~5; $\underline{1}$~~	~~$\underline{4}$; 0~~	1.
	a_2	~~$\underline{6}$; 0~~	3; $\underline{1}$	
	a_3	~~$\underline{6}$; $\underline{4}$~~	$\underline{4}$; $\underline{4}$	
		2.		

Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3



P_2

		b_1	b_2
P_1	a_1	5; $\underline{1}$	$\underline{4}$; 0
	a_2	$\underline{6}$; 0	3; $\underline{1}$
	a_3	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$

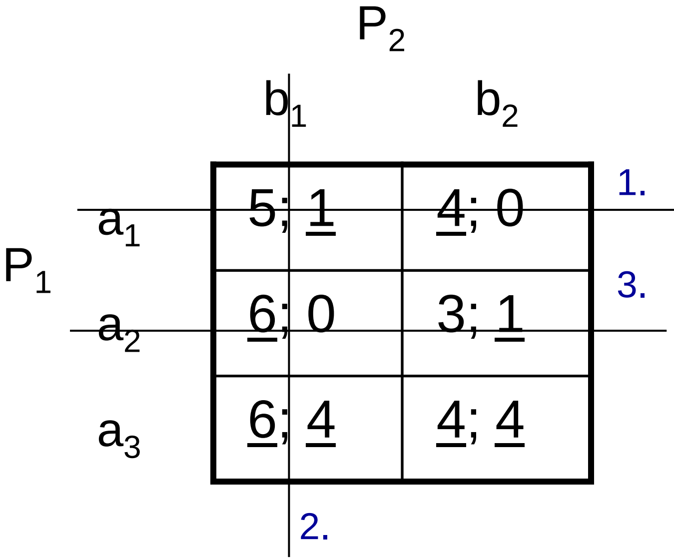
Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$



		P_2	
		b_1	b_2
P_1	a_1	5; $\underline{1}$	$\underline{4}$; 0
	a_2	$\underline{6}$; 0	3; $\underline{1}$
	a_3	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$

Does the order of deletion affect the set of strategies that remains in the end ?

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{P_2} \\
 & & b_1 & b_2 \\
\hline
 & a_1 & 5;\ \underline{1} & \underline{4};\ 0 \\
\hline
P_1 & a_2 & \underline{6};\ 0 & 3;\ \underline{1} \\
\hline
 & a_3 & \underline{6};\ \underline{4} & \underline{4};\ \underline{4} \\
\hline
\end{array}$$

ISD: NO
IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$

Does the order of deletion affect the set of strategies that remains in the end ?

		P_2		
		b_1	b_2	
	a_1	5; $\underline{1}$	$\underline{4}$; 0	
P_1	~~a_2~~	~~$\underline{6}$; 0~~	~~3; $\underline{1}$~~	1.
	a_3	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$	

ISD: NO
IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$

1. a_2 is WDed by a_3

Does the order of deletion affect the set of strategies that remains in the end ?

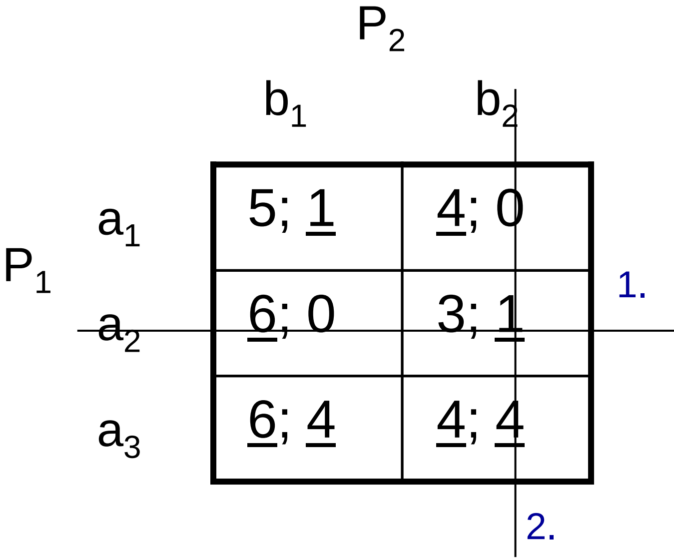
ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2=\{b_2\}$

1. a_2 is WDed by a_3
2. b_2 is WDed by b_1



		P_2	
		b_1	b_2
P_1	a_1	$5; \underline{1}$	$\underline{4}; 0$
	a_2	$\underline{6}; 0$	$3; \underline{1}$
	a_3	$\underline{6}; \underline{4}$	$\underline{4}; \underline{4}$

Does the order of deletion affect the set of strategies that remains in the end ?

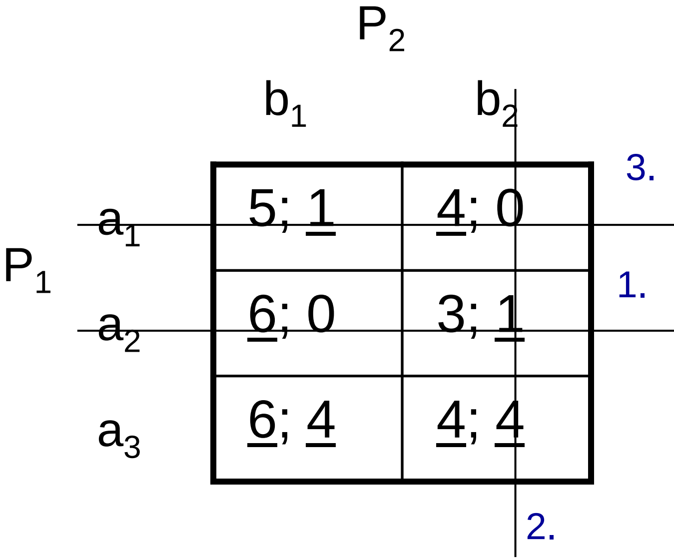
ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$

1. a_2 is WDed by a_3
2. b_2 is WDed by b_1
3. a_1 is SDed by a_3



		P_2	
		b_1	b_2
P_1	a_1	$5; \underline{1}$	$\underline{4}; 0$
	a_2	$\underline{6}; 0$	$3; \underline{1}$
	a_3	$\underline{6}; \underline{4}$	$\underline{4}; \underline{4}$

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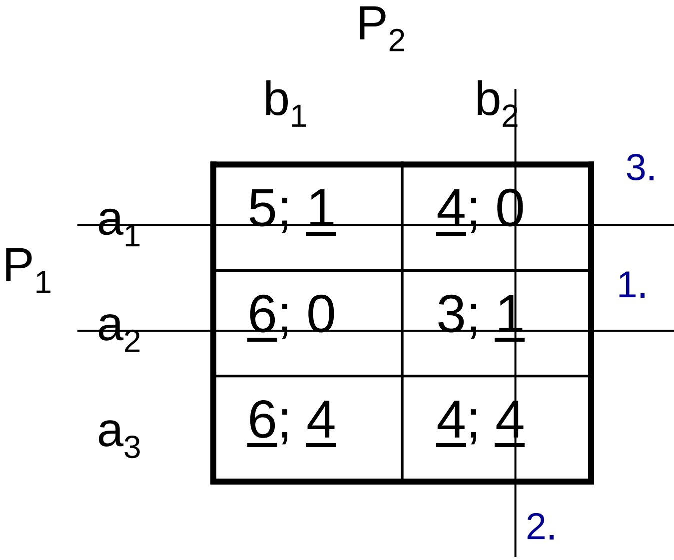

		P$_2$	
		b$_1$	b$_2$
P$_1$	a$_1$	5; $\underline{1}$	$\underline{4}$; 0
	a$_2$	$\underline{6}$; 0	3; $\underline{1}$
	a$_3$	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$

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IWD: YES, the order of deletion can affect the outcome

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2. b_2 is WDed by b_1
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IWD: YES, the order of deletion can affect the outcome

		P_2	
		b_1	b_2
P_1	a_1	$5; \underline{1}$	$\underline{4}; 0$
	a_2	$\underline{6}; 0$	$3; \underline{1}$
	a_3	$\underline{6}; \underline{4}$	$\underline{4}; \underline{4}$

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$

1. a_2 is WDed by a_3
2. b_2 is WDed by b_1
3. a_1 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_1\}$

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3. a_2 is SDed by a_3

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1. a_2 is WDed by a_3
2. b_2 is WDed by b_1
3. a_1 is SDed by a_3

$I_1=\{a_3\}$, $I_2=\{b_1\}$

		P_2	
		b_1	b_2
	~~a_1~~	~~$5; \underline{1}$~~	~~$\underline{4}; 0$~~
P_1	a_2	$\underline{6}; 0$	$3; \underline{1}$
	a_3	$\underline{6}; \underline{4}$	$\underline{4}; \underline{4}$

1.

1. a_1 is WDed by a_3

Does the order of deletion affect the set of strategies that remains in the end ?

		P_2		
		b_1	b_2	
P_1	~~a_1~~	~~5; $\underline{1}$~~	~~$\underline{4}$; 0~~	1.
	~~a_2~~	~~$\underline{6}$; 0~~	~~3; $\underline{1}$~~	2.
	a_3	$\underline{6}$; $\underline{4}$	$\underline{4}$; $\underline{4}$	

ISD: NO

IWD: YES, the order of deletion can affect the outcome

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2=\{b_2\}$

1. a_2 is WDed by a_3
2. b_2 is WDed by b_1
3. a_1 is SDed by a_3

$I_1=\{a_3\}$, $I_2=\{b_1\}$

1. a_1 is WDed by a_3
2. a_2 is WDed by a_3

Does the order of deletion affect the set of strategies that remains in the end ?

ISD: NO
IWD: YES, the order of deletion can affect the outcome

		P_2		
		b_1	b_2	
	~~a_1~~	~~$5; \underline{1}$~~	~~$\underline{4}; 0$~~	1.
P_1	~~a_2~~	~~$\underline{6}; 0$~~	~~$3; \underline{1}$~~	2.
	a_3	$\underline{6}; \underline{4}$	$\underline{4}; \underline{4}$	

1. a_1 is WDed by a_3
2. b_1 is WDed by b_2
3. a_2 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_2\}$

1. a_2 is WDed by a_3
2. b_2 is WDed by b_1
3. a_1 is SDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_1\}$

1. a_1 is WDed by a_3
2. a_2 is WDed by a_3

$I_1=\{a_3\}$, $I_2= \{b_1, b_2\}$

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

		P_2	
		b_1	b_2
P_1	a_1	1 ; 4	2 ; 3
	a_2	3 ; 2	4 ; 1

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

		P_2	
		b_1	b_2
P_1	a_1	1 ; 4	2 ; 3
	a_2	3 ; 2	4 ; 1

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

		P_2	
		b_1	b_2
P_1	a_1	1 ; 4	2 ; 3
	a_2	$\underline{3}$; 2	$\underline{4}$; 1

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

		P_2	
		b_1	b_2
P_1	a_1	1 ; 4	2 ; 3
	a_2	3 ; 2	4 ; 1

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

$$\begin{array}{cc|c|c}
 & & \multicolumn{2}{c}{P_2} \\
 & & b_1 & b_2 \\
\hline
 & a_1 & 1 ; \underline{4} & 2 ; 3 \\
\hline
P_1 & a_2 & \underline{3} ; \underline{2} & \underline{4} ; 1 \\
\hline
\end{array}$$

Solution concept

Imperfect-Information Games

Nash Equlibrium -
mutually the best response

1.Mark the best response of P_1:
For each strategy of P_2 (column) mark ALL highest payoffs of P_1

2.Mark the best response of P_2:
For each strategy of P_1 (row) mark ALL highest payoffs of P_2

3. Mutually the best response (cell with 2 marked numbers) = NE

P_2

		b_1	b_2
P_1	a_1	1 ; $\underline{4}$	2 ; 3
	a_2	**$\underline{3}$; $\underline{2}$**	$\underline{4}$; 1

NE: $P_1 \rightarrow a_2$; $P_2 \rightarrow b_1$
(a_2; b_1)
Payoffs: (3;2)

Nash Equilibrium

The strategy combination s^* **is a Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate.

Formally, for each player *i* $U_i\,(s_i{*}, s_{-i}{}^{*}) \geq U_i\,(s_i{'}, s_{-i}{}^{*})$ for every strategy $s_i{'}$.

In a **Nash equilibrium**, each player’s strategy choice is a **best response** to the strategies actually played by other players.

Nash Equilibrium

The strategy combination s^* **is a Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate.

Formally, for each player *i* $U_i\,(s_i^*, s_{-i}^*) \geq U_i\,(s_i', s_{-i}^*)$ for every strategy s_i' .

In a **Nash equilibrium**, each player’s strategy choice is a **best response** to the strategies actually played by other players.

Solution concept

Rationalizability

Nash equilibrium

ANY beliefs that a strategy is the best response

CORRECT beliefs that a strategy is the best response

Mixed strategies

Players may be indifferent between two moves and instead of picking one action or the other they can simply flip a coin or otherwise “randomize” over actions.

Definition. A **mixed strategy** is a probability distribution over actions.

Nash Equilibrium

The strategy profile s^* **is a Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate.

Formally, for each player *i* $U_i(s_i^*, s_{-i}^*) \geq U_i(s_i', s_{-i}^*)$ for every strategy s_i' .

Nash Equilibrium in mixed strategies

The mixed or pure strategy profile σ^* **is a Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate.

Formally, for each player *i* $EU_i(\sigma_i^*, \sigma_{-i}^*) \geq EU_i(\sigma_i', \sigma_{-i}^*)$ for every strategy σ_i' .

Nash Equilibrium in mixed strategies

The mixed or pure strategy profile σ^* **is a Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate. Formally, for each player *i* $EU_i\,(\sigma_i{}^*, \sigma_{-i}{}^*) \geq EU_i\,(\sigma_i', \sigma_{-i}{}^*)$ for every strategy $\sigma_i{}'$.

Nash Equilibrium

Strict

Weak

$EU_i\,(\sigma_i{}^*, \sigma_{-i}{}^*) > EU_i\,(\sigma_i', \sigma_{-i}{}^*)$

$EU_i\,(\sigma_i{}^*, \sigma_{-i}{}^*) \geq EU_i\,(\sigma_i', \sigma_{-i}{}^*)$

Important points:

Rational person is randomizing between actions only if he is indifferent between these actions.=> We must find beliefs such that actions over which he is mixing all yields him the *same expected payoffs*.

Whenever a player is willing to play one mixed strategy between two or more actions, he is willing to play any such mixed strategy. But exactly which mixed strategy a player plays in NE is determined by what would make the OTHER player indifferent, not what the first player prefers.

Fundamental theorem of mixed strategies:

For given beliefs, a mixed strategy is optimal for a player if and only if each action he plays with positive probability is optimal given those beliefs.

=> players mix only between rationalizable actions

It is not true, that a mixed strategy is always rationalizable IF it mixes over rationalizable actions

=> always check if there is no other action which yields strictly better expected payoff than the mixed strategy in consideration. If such action exists => the mixed strategy is not rationalizable and player would not mix.

Find all NE.

		P_2	
		b_1	b_2
P_1	a_1	0; 1	1; 0
	a_2	1; 0	0; 1

Find all NE.

		P_2		
		b_1	b_2	
P_1	a_1	0; $\underline{1}$	$\underline{1}$; 0	p
	a_2	$\underline{1}$; 0	0; $\underline{1}$	1-p
		q	1-q	

Pure strategy NE: None
Mixed strategy NE:

$EU_1(a_1)=EU_1(a_2)$
$1-q=q$ ◂ => $q=1/2$

$EU_2(b_1)=EU_2(b_2)$
$p=1-p$ => $p=1/2$

=> Mixed strategy NE: $(p*a_1+(1-p)*a_2 ; q*b_1+(1-q)*b_2)$
$(½*a_1 + ½*a_2 ; ½*b_1 + ½*b_2)$

Payoffs: (1/2; 1/2)

Every game with a *finite number of strategies* has a mixed strategy NE.

Note: here we consider a pure strategy as a degenerated mixed strategy

Find all NE.

			P_2	
		b_1	b_2	b_3
	a_1	3; 2	0; 1	1; 1
P_1	a_2	2; 1	2; 2	4; 0
	a_3	0; 1	3; 3	0; 0

Find <u>all</u> NE.

1. b_3 is SDed by b_1
   a_2 is rationalizable

$R_1=\{a_1, a_2, a_3\}$, $R_2=\{b_1,b_2\}$

NE: (a_1, b_1), (a_3, b_2)

$EU_1(a_1)=3q$
$EU_1(a_2)=2$
$EU_1(a_3)=3-3q$

			P_2			
		b_1	b_2	b_3		
	a_1	$\underline{3}; \underline{2}$	$0; 1$	$1; 1$	p	
P_1	a_2	$2; 1$	$2; \underline{2}$	$\underline{4}; 0$	1-p	m
	a_3	$0; 1$	$\underline{3}; \underline{3}$	$0; 0$		1-m
		q	1-q	1.		

$EU_1(a_1)= EU_1(a_2) \Rightarrow q=2/3$ $EU_1(a_3)=3-3*2/3=1<2$ –mix between a_1 and a_2
$EU_1(a_2)= EU_1(a_3) \Rightarrow q=1/3$ $EU_1(a_1)=3*1/3=1<2$ –mix between a_2 and a_3
$EU_1(a_1)= EU_1(a_3) \Rightarrow q=1/2$ –mixing b/n a_1 and a_3 is not rationalizable

$q=2/3$
$EU_2(b_1)=2p+1-p=p+1$
$EU_2(b_2)=p+2-2p=2-p$
$EU_2(b_1)= EU_2(b_2) \Rightarrow p=1/2$

$q=1/3$
$EU_2(b_1)=1$
$EU_2(b_2)=2m+3-3m=3-m$
$EU_2(b_1)= EU_2(b_2) \Rightarrow m=2 >1 \Rightarrow$ no mixing

Mixed NE: $(1/2a_1+1/2a_2; 2/3b_1+1/3b_2)$, payoffs $(2;3/2)$
NOTE: payoffs are just Expected Utilities:
for P1 payoff is : $EU_1(a_2)=EU_1(a_1)=2$
for P2 payoff is : $EU_2(b_1)=EU_2(b_2)=3/2$

Find all NE.

			P_2	
		b_1	b_2	b_3
	a_1	2; 2	0; 1	5; 0
P_1	a_2	1; 0	3; 3	0; 0
	a_3	0; 5	0; 0	4; 4

Find all NE.

1.a_3 is SDed by a_1
2.b_3 is SDed by b_1

$R_1=\{a_1,a_2\}$, $R_2=\{b_1,b_2\}$

NE: (a_1, b_1), (a_2, b_2)

$EU_1(a_1)=2q$
$EU_1(a_2)=3-2q$
$EU_1(a_1)= EU_1(a_2) \Rightarrow q=3/4$

$EU_2(b_1)=2p$
$EU_2(b_2)=3-2p$
$EU_2(b_1)= EU_2(b_2) \Rightarrow p=3/4$

Mixed NE: $(3/4a_1+1/4a_2; 3/4b_1+1/4b_2)$,
payoffs $(3/2;3/2)$

Payoffs: for P1 payoff is :
$EU_1(a_2)=EU_1(a_1)=3/2$
for P2 payoff is :
$EU_2(b_1)=EU_2(b_2)=3/2$

$$\begin{array}{cc|c|c|c|c}
 & & & P_2 & & \\
 & & b_1 & b_2 & \cancel{b_3} & \\
\hline
 & a_1 & \underline{2};\ \underline{2} & 0;\ 1 & \cancel{\underline{5};\ 0} & p \\
\hline
P_1 & a_2 & 1;\ 0 & \underline{3};\ \underline{3} & \cancel{0;\ 0} & 1-p \\
\hline
 & \cancel{a_3} & \cancel{0;\ \underline{5}} & \cancel{0;\ 0} & \cancel{4;\ 4} & 1. \\
\hline
 & & q & 1-q & 2. & \\
\end{array}$$

Symmetric Nash Equilibrium

In symmetric games mixed strategy NE is symmetric, e.i. both players plays the same strategy.

Symmetric game:

- Players’s set of actions are the same
- $U_1(a_i,a_j)=U_2(a_j,a_i)$ for each action pair (a_i,a_j)

		P_2	
		a_1	a_2
P_1	a_1	w; w	x; y
	a_2	y; x	z; z

Homework 2.1 : Consider the game

P_2

P_1

	b_1	b_2	b_3	b_4
a_1	6; 2	0; 2	6; 5	-1; -2
a_2	6; 5	4; 2	2; 2	0; -1
a_3	1; 2	0; 6	2; 3	2; -3
a_4	0;-1	-4; 0	0; 1	-6; 0

1.List, for each player, the set of (pure) strategies consistent with rationalizability

2. List, for each player, the set of (pure) strategies remaining after the iterated deletion of weakly dominated strategies.

3 Find pure-strategy Nash equilibria and their generated payoffs in this game.

4. Find mixed-strategy Nash equilibria and their generated payoffs in this game.

Homework 2.1 : Consider the game

P_2

		b_1	b_2	b_3	b_4
P_1	a_1	6; 2	0; 2	6; 5	-1; -2
	a_2	6; 5	4; 2	2; 2	0; -1
	a_3	1; 2	0; 6	2; 3	2; -3
	a_4	0;-1	-4; 0	0; 1	-6; 0

1.List, for each player, the set of (pure) strategies consistent with rationalizability

$R_1=\{a_1,a_2\}$; $R_2=\{b_1, b_3\}$

2. List, for each player, the set of (pure) strategies remaining after the iterated deletion of weakly dominated strategies.

$I_1=\{a_1\}$; $I_2=\{ b_3\}$

3 Find pure-strategy Nash equilibria and their generated payoffs in this game.

NE_1: $(a_1; b_3)$ payoffs (6; 5)
NE_2: $(a_2; b_1)$ payoffs (6; 5)

4. Find mixed-strategy Nash equilibria and their generated payoffs in this game.

NE_3: $(0.5a_1+0.5a_2; b_1)$ payoffs (6; 3.5)

Homework 2.2 : Find all NE in the game

		P_2			
		b_1	b_2	b_3	b_4
P_1	a_1	2; 2	10; 0	0; 0	0; 0
	a_2	0;10	5; 5	0; 0	0; 0
	a_3	0; 0	0; 0	10; 1	0; 0
	a_4	0; 0	0; 0	0; 0	1; 10

Homework 2.2 : Find <u>all</u> NE in the game

1.a_2 is not rationalizable (prove)
2.b_2 is not rationalizable (prove)

$R_1=\{a_1,a_3,a_4\}$, $R_2=\{b_1,b_3,b_4\}$

NE: (a_1, b_1), (a_3, b_3), (a_4, b_4)

Mixed NE:

$(5/16a_1+10/16a_3+1/16a_4; 5/16b_1+1/16b_3+10/16b_4)$,
payoffs (5/8;5/8)

$(1/3a_1+2/3a_3; 5/6b_1+1/6b_3)$,
payoffs (5/3;2/3)

$(5/6a_1+1/6a_4; 1/3b_1+2/3b_4)$,
payoffs (2/3;5/3)

$(10/11a_3+1/11a_4; 1/11b_3+10/11b_4)$,
payoffs (10/11;10/11)

		P_2			
		b_1	b_2	b_3	b_4
P_1	a_1	$\underline{2}; \underline{2}$	$\underline{10}; 0$	0; 0	0; 0
	a_2	$0;\underline{10}$	5; 5	0; 0	0; 0
	a_3	0; 0	0; 0	$\underline{10}; \underline{1}$	0; 0
	a_4	0; 0	0; 0	0; 0	$\underline{1}; \underline{10}$

1.

2.