

NMMO401 CONTINUUM MECHANICS

SYLLABUS

VÍT PRŮŠA

Knowledge of topics/notions shown in *italics* is not required for passing the exam. Since the class size is usually small, the covered topics might be customized according to the knowledge level and interest of the enrolled students.

- Preliminaries.
 - Linear algebra.
 - * Scalar product, vector product, mixed product, tensor product. Transposed matrix.
 - * Tensors. *Inertia tensor as an example of a tensorial quantity in mechanics.*
 - * Cofactor matrix $\text{cof } \mathbb{A}$ and determinant $\det \mathbb{A}$. Geometrical interpretation.
 - * Cayley–Hamilton theorem, characteristic polynomial, eigenvectors, eigenvalues.
 - * Trace of a matrix.
 - * Invariants of a matrix and their relation to the eigenvalues and the mixed product.
 - * Properties of proper orthogonal matrices, angular velocity.
 - * Polar decomposition. Geometrical interpretation.
 - * Spectral decomposition.
 - Elementary calculus.
 - * Matrix functions. Exponential of a matrix. *Skew-symmetric matrices as infinitesimal generators of the group of rotations.*
 - * Representation theorem for scalar valued isotropic tensorial functions and tensor valued isotropic tensorial functions.
 - * Gâteaux derivative, Fréchet derivative. Derivatives of the invariants of a matrix.
 - * Operators ∇ , div and rot for scalar and vector fields. Operators div and rot for tensor fields. Abstract definitions and formulae in Cartesian coordinate system. Identities in tensor calculus.
 - Line, surface and volume integrals.
 - * Line integral of a scalar valued function $\int_{\gamma} \varphi \, d\mathbf{X}$, line integral of a vector valued function $\int_{\gamma} \mathbf{v} \bullet d\mathbf{X}$.
 - * Surface integral of a scalar valued function $\int_S \varphi \, dS$, surface integral of a vector valued function $\int_S \mathbf{v} \bullet d\mathbf{S}$, surface Jacobian.
 - * Volume integral, Jacobian matrix.
 - Stokes theorem and its consequences.
 - * Green identities.
 - * *Potential vector field, path independent integrals, curl free vector fields. Characterisation of potential vector fields.*
 - * *Helmholtz decomposition, $\mathbf{v} = -\nabla\varphi + \text{rot } \mathbb{A}$.*
 - * *Korn equality.*
 - Elementary concepts in classical physics.
 - * Newton laws.
 - * Galilei principle of relativity, non-inertial reference frame.
 - * Fictitious forces (Euler, centrifugal, Coriolis).
- Kinematics of continuous medium.
 - Basic concepts.
 - * Notion of continuous body. Abstract body, place, configuration.
 - * Reference and current configuration. Lagrangian and Eulerian description.
 - * Deformation/motion χ .
 - * Local and *global* invertibility of the motion/deformation.
 - * Deformation gradient \mathbb{F} and its geometrical interpretation. Polar decomposition of the deformation gradient and its geometrical interpretation.
 - * Deformation gradient for simple shear, *pure bending and inflation of a hollow cylinder.*
 - * Displacement \mathbf{U} .
 - * Deformation of infinitesimal line, surface and volume elements. Concept of isochoric motion.
 - * Lagrangian velocity field \mathbf{V} , Eulerian velocity field \mathbf{v} . Material time derivative $\frac{d}{dt}$ of Eulerian quantities.
 - * Streamlines and pathlines (trajectories).
 - * *Relative deformation/motion.*
 - * Spatial velocity gradient \mathbb{L} , its symmetric \mathbb{D} and skew-symmetric part \mathbb{W} . *Relation of \mathbb{D} and \mathbb{W} to the time derivatives of the relative deformation/motion.*

- Strain measures.
 - * Left and right Cauchy–Green tensor, \mathbb{B} and \mathbb{C} .
 - * Green–Saint-Venant strain tensor \mathbb{E} , Euler–Almansi strain tensor \mathbb{e} . Geometrical interpretation.
 - * Linearised strain \mathbb{e} .
- *Compatibility conditions for linearised strain \mathbb{e} .*
- Rate quantities.
 - * Rate of change of Green–Saint-Venant strain, rate of change of Euler–Almansi strain and their relation to \mathbb{D} .
 - * Rate of change of infinitesimal line, surface and volume elements. Divergence of Eulerian velocity field and its relation to the change of volume.
- Kinematics of moving surfaces.
 - * Lagrange criterion for material surfaces.
 - * *Formula for the projection of the velocity of the surface to the direction parallel to the normal to the surface.*
- Reynolds transport theorem. *Reynolds transport theorem in the presence of discontinuities.*
- Dynamics of continuous medium.
 - Mechanics.
 - * Balance laws for continuous medium as counterparts of the classical laws of Newtonian physics of point particles.
 - * Concept of contact/surface forces. Existence of the Cauchy stress tensor \mathbb{T} (tetrahedron argument).
 - * Pure tension, pure compression, tensile stress, shear stress.
 - * Balance of mass, linear momentum and angular momentum in Eulerian description.
 - * Balance of angular momentum and its implications with respect to the symmetry of the Cauchy stress tensor.
 - * Balance of mass, linear momentum and angular momentum in Lagrangian description.
 - * First Piola–Kirchhoff tensor \mathbb{T}_R . Piola transformation.
 - * Formulation of boundary value problems in Eulerian and Lagrangian description, transformation of traction boundary conditions from the current to the reference configuration.
 - Elementary concepts in thermodynamics of continuous medium.
 - * Internal energy, heat flux.
 - * Balance of total energy in Eulerian and Lagrangian description.
 - * Balance of internal energy in Eulerian and Lagrangian description.
 - * Referential heat flux.
 - Boundary conditions.
 - Geometrical linearisation. Incompressibility condition in the linearised setting. Specification of the boundary conditions in the linearised setting.
 - *Balance laws in the presence of discontinuities.*
- Simple constitutive relations.
 - Pressure and thermodynamic pressure. Derivation of compressible and incompressible Navier–Stokes fluid model via the representation theorem for tensor valued isotropic tensorial functions.
 - Cauchy elastic material. Derivation via the representation theorem for tensor valued isotropic tensorial functions.
 - Physical units, dimensionless quantities, Reynolds number.
- *Simple problems in mechanics of continuous medium.*
 - *Archimedes law.*
 - *Deformation of a cylinder (linearised elasticity). Hooke law.*
 - *Inflation of a hollow cylinder made of an incompressible isotropic elastic solid. (Comparison of linearised theory and fully nonlinear theory.)*
 - *Isothermal atmosphere versus swimming pool. (Difference between the notion of the pressure in the case of compressible and incompressible Navier–Stokes fluid model.)*
 - *Stokes formula via dimensional analysis.*
 - *Waves in compressible Navier–Stokes fluid.*
 - *Waves in linearised isotropic elastic solid.*
 - *Stability of the rest state of the incompressible Navier–Stokes fluid.*
 - *Pressure driven flow of incompressible Navier–Stokes fluid in a pipe of circular cross section.*