Inquisitive Semantics I

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Inquisitive Semantics

- a framework for a logical analysis of questions
- developed around 2009 by Jeroen Groenendijk, Floris Roelofsen, and Ivano Ciardelli
- https://projects.illc.uva.nl/inquisitivesemantics/

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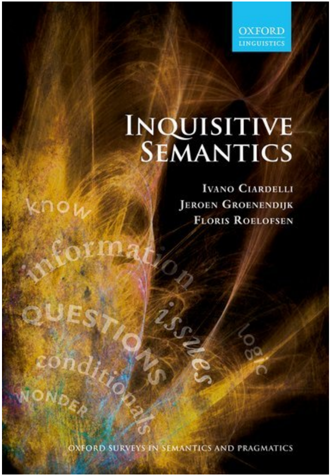
OXFORD
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IVANO CIARDELLI
JEROEN GROENENDIJK
FLORIS ROELOFSEN
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information
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WONDER
OXFORD SURVEYS IN SEMANTICS AND PRAGMATICS

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The basic logical picture

- logic is concerned with (formal) validity of arguments
- an argument is a structure

$$A_1, \ldots, A_n / B$$

where $A_1, \ldots, A_n$, B are declarative sentences

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Logic of imperatives

- Buy bread and butter! / Buy bread!
- Invite all Peter's friends!, John is a Peter's friend. / Invite John!
- Send the letter! / Send the letter or burn it!
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The standard propositional language

$$\alpha := p \mid \bot \mid \alpha \to \alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha.$$

$$\neg\alpha =_{def} \alpha \to \bot$$
$$\alpha \leftrightarrow \beta =_{def} (\alpha \to \beta) \wedge (\beta \to \alpha)$$

Intuitionistic logic

- $\alpha \to (\beta \to \alpha)$,
- $(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$,
- $(\alpha \wedge \beta) \to \alpha$,
- $(\alpha \wedge \beta) \to \beta$,
- $\alpha \to (\alpha \vee \beta)$,
- $\beta \to (\alpha \vee \beta)$,
- $(\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$,
- $\perp \to \alpha$.

- $\alpha, \alpha \to \beta / \beta$.

Logic of problems

Kolmogorov, A. (1932). Zur Deutung der intuitionistischen Logik, *Mathematische Zeitschrift*, 35, 58–65.

- while classical logic captures the logical relations among statements, intuitionistic logic captures logical relations among problems

Examples of problems

- to draw a circle through three given points
- to find a root of the equation $ax^2 + bx + c = 0$
- provided that one root of the equation $ax^2 + bx + c = 0$ is given, to find the other root
- to express the number π as a ratio m/n

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Atomic and complex problems

if φ and ψ are problems, then

- $\varphi \wedge \psi$ is the problem to solve both problems φ and ψ;
- $\varphi \vee \psi$ is the problem to solve at least one of the problems φ and ψ;
- $\varphi \to \psi$ is the problem to reduce the solution of ψ to the solution of φ;
- $\neg\varphi$ is the problem to obtain a contradiction, given a solution of φ.

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Validity in the logic of problems

- a declarative sentence is logically valid iff it is true on the basis of its form (all sentences of the same form are true)
- a problem is logically valid iff it is solvable on the basis of its form (all problems of the same form are solvable)
- φ is valid in the logic of problems iff there is a uniform solution of the problems of the form φ
- $\varphi_1, \ldots, \varphi_n / \psi$ is valid if there is a uniform method reducing the solution of ψ to the solution of $\varphi_1, \ldots, \varphi_n$

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Examples of validities

- all principles of intuitionistic logic are valid in this sense
- for example the following are valid:
  - $p \to q, q \to r / p \to r$
  - $(p \to q) \to ((p \wedge r) \to (q \wedge r))$

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The principle of excluded middle

The validity of $p \vee \neg p$ would mean that there is a general method that gives us for any problem either a solution of the problem, or a derivation of a contradiction from the assumption of such a solution.

An example due to Ivano Ciardelli

- a certain disease may give rise to two symptoms: S_1, S_2
- a hospital's protocol:

> if a patient presents symptom S_2, the treatment is always prescribed; if the patient only presents symptom S_1, the treatment is prescribed just in case the patient is in good physical condition; if not, the risk associated with the treatment outweigh the benefits, and the treatment is not prescribed

Types of information

Examples of types of information:

- patient's symptoms ($S_1, S_2, \ldots$)
- patient's conditions (good, bad)
- treatment (prescribed, not prescribed)

Logic of dependencies among types of information

In the given context, a specific *relation* holds between different *types* of information: complete information about a patient's symptoms, combined with complete information about the patient's conditions, is guaranteed to yield complete information about whether the treatment should be administered. We may say that, given the protocol, information of type *'symptoms'*, together with information of type *'conditions'*, *yields* information of type *'treatment'*.

Validity of arguments

Let A, B be types of information. A/B is valid (i.e. the type B is dependent on the type A) iff any piece of information of the type A entails some piece of information of the type B.

Logic of questions

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Making sense of arguments with questions I
Inferential Erotetic Logic

- P1 Mary is Peter's mother.
- P2 If Mary is Peter's mother, then John is Peter's father or George is Peter's father.
- C Who is Peter's father: John or George?

Making sense of arguments with questions II
Inquisitive Logic

$S1, S2$ are statements and $Q1, Q2$ questions

an argument	its intended interpretation
$S1/S2$	$S1$ implies $S2$
$Q1/S2$	$Q1$ presupposes $S2$
$S1/Q2$	$S1$ resolves $Q2$
$Q1/Q2$	any information that resolves $Q1$ resolves also $Q2$

Examples

(a) The statements *if Mary is Peter's mother, then John is not Peter's father* and *John is Peter's father* together resolve the question *whether Mary is Peter's mother*.

(b) The question *who is Peter's father: John or George?* pressuposes that *John or Georg is Peter's father*.

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Difference between IEL and InqL

A Who is Peter's father: John or George?

B John or George is Peter's father.

- According to InqL A entails B but B does not entail A
- According to IEL B entails A but A does not entail B

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More complex examples

(c) Valid: Assume that *if today is not Monday and Mary is at home, then John is in the library* and *if John is in the library and Mary is not at home, then it is Monday*. Then any information that resolves the question *whether John is in the library* resolves also the conditional question *whether Mary at home if it is not Monday*.

(d) Invalid: Any information that resolves the conditional questions *whether John is in the library, if Mary is at home* and *whether Mary is at home, if it is not Monday*, resolves also the question *whether John is in the library, if it is not Monday*.

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Formalization via inquisitive disjunction

- disjunctive questions: *whether A or B*.
- polar questions: *whether A* = *whether A or not A*.

A nonstandard propositional language

- InqB is a logic for a basic propositional language with one additional operator: inquisitive disjunction $\mathbb{V}$;
- $\mathbb{V}$ is a question-generating operator: $\varphi \mathbb{V} \psi$ is interpreted as the question *whether* φ *or* ψ;
- $?\varphi =_{def} \varphi \mathbb{V} \neg\varphi$ (the question *whether* φ)

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The standard language of propositional (intuitionistic) logic

$$\varphi := p \mid \bot \mid \varphi \to \varphi \mid \varphi \wedge \varphi \mid \varphi \mathbin{\vee\!\!\!\vee} \varphi.$$

$$\neg\varphi =_{def} \varphi \to \bot$$

$$\varphi \vee \psi =_{def} \neg(\neg\varphi \wedge \neg\psi)$$

$$\varphi \leftrightarrow \psi =_{def} (\varphi \to \psi) \wedge (\psi \to \varphi)$$

$$?\varphi =_{def} \varphi \vee \neg\varphi$$

$$!\varphi =_{def} \neg\neg\varphi$$

Formalization

(a) Valid: The statements *if today is Monday then it is not Tuesday* and *today is Tuesday* together resolve the question *whether today is Monday*.

$m \to \neg t, t / ?m$,

(i.e. $m \to \neg t, t / m \mathbin{\vee\!\!\!\vee} \neg m$)

Formalization

(b) Invalid: Any information that resolves the (conditional) question *whether John will go swimming today, if it is Monday* resolves also the (unconditional) question *whether John will go swimming today*.

$m \to ?s / ?s$

(i.e. $m \to (s \vee\!\!\!\vee \neg s) / s \vee\!\!\!\vee \neg s$)

Formalization

(c) Valid: Assume that *if today is not Monday and Mary is at home, then John is in the library* and *if John is in the library and Mary is not at home, then it is Monday*. Then any information that resolves the question *whether John is in the library* resolves also the conditional question *whether Mary is at home if it is not Monday*.

$(\neg m \wedge h) \rightarrow l, (l \wedge \neg h) \rightarrow m, ?l / \neg m \rightarrow ?h$

Formalization

(d) Invalid: Any information that resolves the conditional questions *whether John is in the library, if Mary is at home* and *whether Mary is at home, if it is not Monday*, resolves also the question *whether John is in the library, if it is not Monday*.

$h \rightarrow ?l, \neg m \rightarrow ?h / \neg m \rightarrow ?l$

Basic Inquisitive Logic InqB

Intuitionistic logic plus

split $(\alpha \to (\psi \vee\!\!\!\vee \chi)) \to ((\alpha \to \psi) \vee\!\!\!\vee (\alpha \to \chi))$,

rdn $\neg\neg\alpha \to \alpha$,

where α ranges over $\vee\!\!\!\vee$-free formulas.

Propositions expressed by questions

Frege claimed in *The Thought* that a statement (like *Mary is drinking beer*) and the corresponding yes-no question (*Is Mary drinking beer?*) have the same content and differ only in something that is not a part of the content itself.

Frege on questions

An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion.

A problem for Frege's approach

This view seems to be limited to yes-no question.

When we take a disjunctive question (*Is Mary drinking red wine or white wine?*), we cannot identify its content in the same style with the content of any single declarative sentence.

A different approach

Some authors suggest that one can identify the semantic content of a question with the content of a declarative sentence that describes the epistemic presuppositions of the question.

Peliš, M., Majer, O.: Logic of Questions from the Viewpoint of Dynamic Epistemic Logic, in: *The Logica Yearbook 2009*, Peliš, M. (ed.), College Publications, London 2010, pp. 157-172.

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A different approach

According to this view, there is also no substantial difference between declarative and interogative propositions, though we need a rich language, namely a language of a modal epistemic logic, to capture properly the semantic content of questions.

Questions express propositions

In inquisitive semantics questions are regarded as expressing a special kind of propositions.

The meaning of a sentence = its truth conditions

“To understand a proposition means to know what is the case if it is true."

L. Wittgenstein, TLP, 4.024

The sentential meaning of declarative sentences.

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Informative and inquisitive content of sentences

- Inquisitive semantics introduces a richer notion of sentential meaning that is applicable to declarative sentences as well as to questions.
- In inquisitive semantics, sentential meaning is modelled as consisting of an informative part and an inquisitive part.

Informative and inquisitive content of sentences

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Localization of the actual world

- The informative content *info*(A) of a given sentence A can be represented as a set of possible worlds and the sentence provides the information that the actual world is located somewhere in the set.
- The inquisitive content *inq*(A) can be understood as a *request* to locate the actual world more precisely. The request *inq*(A) can be modeled as a set of those nonempty subsets of *info*(A) that contain enough information to settle the request.

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Info(A) can be retrieved from *Inq*(A).

- The request to locate the actual world more precisely in *info*(A) should not a priori exclude any world of *info*(A).
- As a consequence, *info*(A) has to be the union of *inq*(A).

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Propositions as sets of information states

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Truth-functional semantics for classical logic

A truth-functional model: $\mathcal{M} = \langle W, V \rangle$.

The relation of truth:

- p is true in w iff $w \in V(p)$,
- $\perp$ is not true in w,
- $\alpha \to \beta$ is true in w iff α is not true in w or β is true in w
- $\alpha \wedge \beta$ is true in w iff α is true in w and β is true in w

Inquisitive semantics

An inquisitive model: $\mathcal{N} = \langle \mathcal{P}(W), V \rangle$.

The support relation:

$s \vDash p$ iff $s \subseteq V(p)$,

$s \vDash \bot$ iff $s = \emptyset$,

$s \vDash \neg\varphi$ iff for any nonempty $t \subseteq s$, $t \nvDash \varphi$,

$s \vDash \varphi \to \psi$ iff for any $t \subseteq s$, if $t \vDash \varphi$ then $t \vDash \psi$,

$s \vDash \varphi \wedge \psi$ iff $s \vDash \varphi$ and $s \vDash \psi$,

$s \vDash \varphi \vee\!\!\!\vee \psi$ iff $s \vDash \varphi$ or $s \vDash \psi$.

Ontic and informational semantics

- As regards the declarative language the two semantics are equivalent:

  universal truth = universal support
  preservation of truth = preservation of support

- The standard framework is based on ontic objects (possible worlds) and an ontic relation of truth;
- The inquisitive framework is based on informational objects (information states = partial representations of possible worlds) and an informational relation of support.

C. I. Lewis: Implication and the algebra of logic, 1912

One of the important practical uses of implication is the testing of hypotheses whose truth or falsity is problematic. The algebraic [truth-table] implication has no use here. If the hypothesis happens to be false, it implies anything you please... In other words, no proposition could be verified by its logical consequences. If the proposition be false, it has these "consequences" anyway.

Theorem

In every inquisitive model:

(a) *every formula is supported by the empty state,*
(b) *support is downward persistent for all formulas,*
(c) *support of declarative formulas is closed under arbitrary unions,*
(d) *every formula is equivalent to the inquisitive disjunction of a finite set of declarative formulas.*

Examples

a) Jane is in the cinema.
b) Is Peter in the cinema?
c) Is Jane in the cinema with Peter?
d) Peter or Jane is in the cinema.
e) Is Peter or Jane in the cinema?
f) Who is in the cinema: Peter or Jane?
g) If Peter is in the cinema, Jane is also there.
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g) If Peter is in the cinema, Jane is also there.
h) If Peter is in the cinema, is there also Jane?

Medvedev logic of finite problems

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- a formalization of Kolmogorov's ideas
- determines an superintuitionistic logic: Medvedev logic of finite problems

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A remarkable result

Theorem

The schematic fragment of inquisitive logic corresponds to Medvedev logic of finite problems.

An example due to Ivano Ciardelli

- a certain disease may give rise to two symptoms: S_1, S_2
- hospital's protocol:

> if a patient presents symptom S_2, the treatment is always prescribed; if the patient only presents symptom S_1, the treatment is prescribed just in case the patient is in good physical condition; if not, the risk associated with the treatment outweigh the benefits, and the treatment is not prescribed

A formalization of the protocol

The protocol:

- $t \leftrightarrow s_2 \vee (s_1 \wedge g)$

where

- s_1: the patient has symptom S_1
- s_2: the patient has symptom S_2
- g: the patient is in good physical condtion
- t: the treatment is prescribed

Types of information

Examples of types of information:

- patient's symptoms ($S_1, S_2, \ldots$)
- patient's conditions (good, bad)
- treatment (prescribed, not prescribed)

Types of information

Types of information correspond to questions:

- what are the patient's symptoms: $?s_1 \wedge ?s_2$
- whether the patient is in good physical conditions: $?g$
- whether the treatment is prescribed: $?t$

Dependencies among information types correspond to logical relations among questions

$$t \leftrightarrow s_2 \vee (s_1 \wedge g), ?s_1 \wedge ?s_2, ?g \vDash ?t$$